

Annex A

Adopting the notations introduced above, let us first calculate $X^T Y$.

$$\begin{aligned} X^T W(u_1, v_1) Y &= [X_1^T \quad X_2^T] \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \\ &= [X_1^T \quad X_2^T] \cdot \begin{bmatrix} Y_1 \\ 0 \end{bmatrix} \\ &= X_1^T Y_1 \end{aligned}$$

Calculating now $X^T W(u_1, v_1) X$ we find:

$$\begin{aligned} X^T W(u_1, v_1) X &= [X_1^T \quad X_2^T] \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \\ &= [X_1^T \quad X_2^T] \cdot \begin{bmatrix} X_1 \\ 0 \end{bmatrix} \\ &= X_1^T X_1 \end{aligned}$$

Using these two results we have proven that

$$\begin{aligned} (X^T W(u_1, v_1) X)^{-1} (X^T W(u_1, v_1) Y) \\ = (X_1^T X_1)^{-1} X_1^T Y \end{aligned}$$